8

The Foundations of Astrodynamics

SAMUEL HERRICK, *United States*

Astrodynamics is defined in terms of celestial mechanics and of space navigation in its broadest sense: pre-Sputnik, including orbit determination and correction, as well as post-Sputnik, which adds control and optimization.

Basically there are three areas of celestial mechanics:

1. *Mathematical celestial mechanics* is concerned with the existence of solutions to defined and restricted problems in celestial mechanics. It prefers methods that have generality in the sense that they are applicable to other fields of mechanics as well as to a range of problems in celestial mechanics. But these methods tend to be restricted to a given type of problem: e.g., the elegant potential and Hamiltonian methods are limited to conservative and quasi-conservative forces.

2. *Physical celestial mechanics* is concerned primarily with the use of celestial mechanics in the determination of physical constants that are of interest to other areas of physics, especially geophysics and astrophysics.

3. *Astrodynamics,* as we term the third area of celestial mechanics, is greatly interested in physical constants, but also in all other factors that contribute to accurate space navigation, such as integration constants, integration procedures, singularities, and indeterminacies. Astrodynamics makes fundamental use of the general methods of mathematical celestial mechanics, and also of special methods that fit particular real problems. But whereas mathematical celestial mechanics tends to pursue one solution to a conclusion, with maximum use of a particular class of elegant mathematical tools, astrodynamics seeks to develop all possible solutions for purposes of comparison and selection. Mathematical celestial mechanics is concerned with ideal problems involving motion in a theoretically simple framework; astrodynamics is concerned with fitting a theory to observation and to the coordinate systems of the real world, and so is concerned with precession, nutation, aberration, parallax, the reduction of observations (electronic as well as optical), and with all the force fields that are encountered in real problems. General methods and tools (e.g., the method of least squares and Bessel's functions) have often come out of the particular solutions of these real problems.

No "celestial mechanic" devotes himself exclusively to one of these areas, but his heart is likely to be in one of them, and his judgement less than clairvoyant in the others. I shall indicate some of the differences between the areas and their methods in the following discussions of the historical development of astrodynamics before 1940.

My own serious concern with astrodynamics and space navigation began when I was an undergraduate student at Williams College. Four letters from Dr. Robert H. Goddard survive to attest to my plan for graduate study in the area, to Dr. Goddard's encouragement, and to his kindliness in taking time to give it even when his own prospects were bleak. I quote from two paragraphs of one of his letters, dated 15 June 1932:

> . owing to the depression, the rocket project is being discontinued July first, and the matter of its being resumed later is an uncertain one.
>
> I cannot help feeling that a theoretical investigation such as you mention has advantages over experimentation during such times as these .

These letters encouraged me to proceed to graduate study under Armin Otto Leuschner, Russell Tracy Crawford, and C. Donald Shane, at Berkeley, where I developed a thoroughgoing devotion to celestial mechanics as well as to space navigation.

Early History Illuminates the Character of Astrodynamics

Physical celestial mechanics may be said to have begun with Galileo Galilei, Isaac Newton, and the laws of force and gravitation. Astrodynamics and mathematical celestial mechanics, on the other hand, date back at least to Heracleides of Pontus in the fourth century B.C. The Greek invention of epicycles and eccentrics was developed into a system by Apollonius of Perga in the third century and Hipparchus of Alexandria in the second century B.C. It was refined and published by Ptolemy of Alexandria in the second century A.D., and came to be known as the Ptolemaic system. It is generally assumed that the epicycle was discredited by Johannes Kepler some 1500 years later, but in point of fact epicycles have persisted in astrodynamics down to the present day, and have extended their domain into other areas of science under the guise of Fourier series!

Hindsight is a valuable tool in the history of science and serves to illuminate on the one hand the contemporary understanding and acceptance of an idea, and, on the other, its clarity and persistence. The historian of science is likely to emphasize the former; the scientist himself is understandably more interested in the latter.

My own hindsight theory has been presented to my students over the past 20 years, and by them conveyed to others, but for the most part it has remained unpublished in the conventional sense (except in preprints of my reference work *Astrodynamics* [1]). Basically it asserts that history has been unjust to epicycles, and even to Nicolaus Copernicus. (Some historians have gone so far as to say that the system of Copernicus was just as cumbersome as the Ptolemaic system, and that Kepler was the real author of our modern heliocentric theory.)

With hindsight we can see that there are in a planet's motion three kinds of deviation from uniformity that confronted the Greeks and their successors, and required explanation by a "system" such as the Ptolemaic or the Copernican:

1. The annual or Copernican or retrograde deviation, caused by the motion of the Earth around the Sun.

2. The elliptic or Keplerian deviation, explained in simple two-body motion by the discovery that

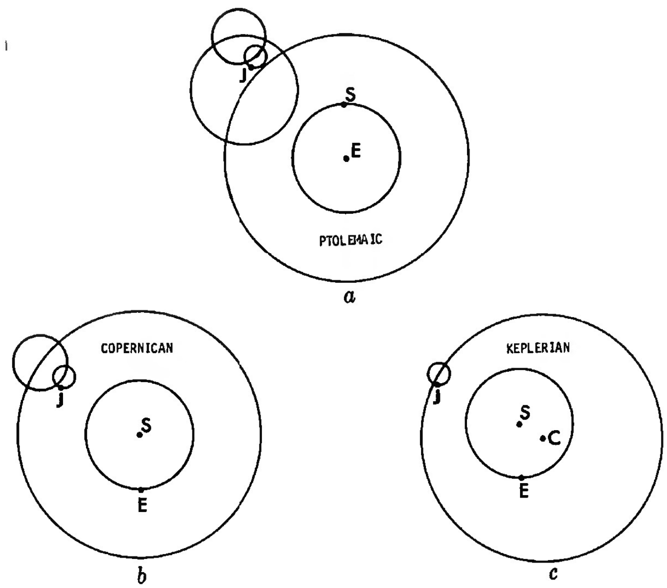

FIGURE 1.—*a*, Ptolemaic, *b*, Copernican, and *c*, Keplerian systems.

the relative orbit of the two bodies is an ellipse or other conic section.

3. The perturbed or Newtonian deviations, caused by the attractions of the planets and their satellites for one another.

The Ptolemaic system explained all of these deviations by geocentric deferents surmounted by epicycles piled upon epicycles (see Figure 1, in which the largest epicycle is the annual one, the second represents the elliptic ones, and the smallest represents the perturbed-deviation epicycles).

Aristarchus of Samos, and later Copernicus, eliminated the first deviation by shifting the center of the system from the Earth to the Sun, but the remaining deviations of the Copernican system still had to be accounted for by epicycles. It is this fact that led to the dictum that the Copernican system is "just about as complicated as the Ptolemaic system." It may have appeared so to contemporary eyes, but in retrospect it is clear that the elimination of the five annual planetary epicycles—that is, one epicycle for each of the five known planets, the total "population" as of that time—was a major simplification of the mechanics of the system, so that Copernicus unquestionably deserves the popular recognition accorded his name.

Kepler accounted for the second class of deviation by his perspicuous laws of planetary motion. It is this fact that has generally been credited with the destruction of the epicycle as a mechanical device.

But it should be recognized that there were perturbed deviations still unaccounted for in the Keplerian system. These deviations are most conspicuous in the motion of the Moon around the Earth, but the observations of Kepler's time were sufficiently accurate to show evidence also of the mutual perturbations of Jupiter and Saturn. When Newton's development of the law of gravitation made it possible to explain these perturbed deviations by mechanical means, the epicycles that had survived Kepler's onslaught were adopted into Newtonian mechanics. As a matter of fact, the basic epicyclic theory re-expanded to include even the elliptic deviations, thus rejecting the Keplerian system in favor of the Copernican system, whose handling of the elliptic terms by systems of epicycles (rechristened "Fourier series") proves to be simpler than the use of expressions in terms of Keplerian ellipses. In a sense this development may be noted as realistic astrodynamic replacement for a theoretical mathematical formulation.

We may note that Fourier series, with arguments that are multiples of a single angle, are less flexible than the original "astrodynamic" concept of epicycles, in which noncommensurate arguments are used: consider, for example, the representation of the geocentric motion of Venus, assuming that Venus and Earth are both travelling in circular heliocentric orbits. The Ptolemaic development would require only one epicycle; the Fourier development would require a theoretically infinite number of terms or epicycles. In modern perturbation theory we actually take account of the original epicyclic concept by combining several Fourier series that have arguments based upon different angular variables.

Astrodynamics Illuminated by Modern Treatment of Parallax

Recent developments in the treatment of geocentric parallax illustrate the importance to astrodynamics of physically real reference systems, and of the reduction of observations, as contrasted with developments in mathematical celestial mechanics, in which the reference system is idealized and observations are only theoretically taken into account.

Figure 2 shows how geocentric parallax enters into the observations. The position of the Sun is designated by *S*, that of the observed object by the

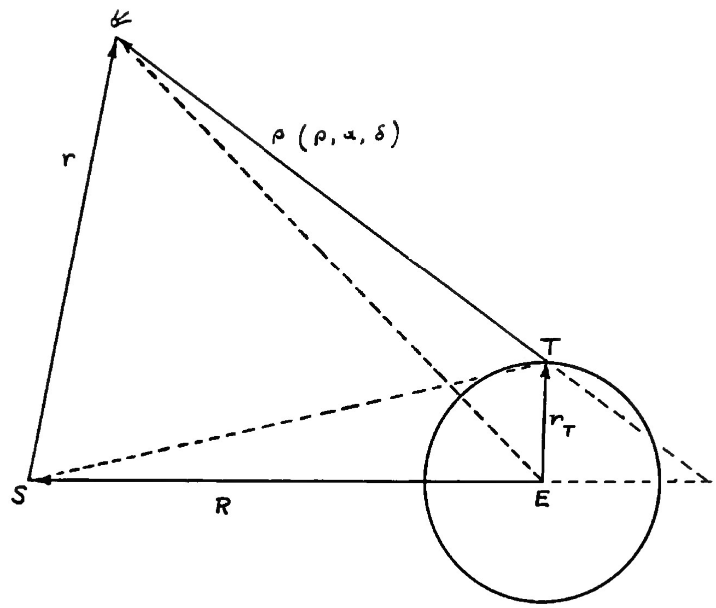

FIGURE 2.—Effect of geocentric parallax on observations.

cometary symbol "☄", the center of the Earth by *E*, and the observer by *T* (for Greek *topos*, place, and for the adjective topocentric). The dynamical position of ☄ is defined by the vector r (i.e., the line segment of *S*☄). The position of the Sun referred to the center of the Earth is specified conventionally by the "solar coordinates" that are given in astronomical almanacs, i.e., by the vector R (in the figure, *ES*). The geocentric position of the observer is specified by r_T (*ET*). Finally the topocentric position of the comet is specified by the vector ρ, which represents the topocentric distance (today the "range") ρ, right ascension α, and declination δ.

Classically the topocentric right ascension and declination are corrected for geocentric parallax to what they would have been had the observation been made from the center of the Earth, so that we have a single triangle relating *E*, *S*, and ☄. In some problems there is still justification for such a procedure, but in preliminary orbit calculations based upon observations of α and δ the parallax can be calculated only after a first approximation has given a value of ρ. Successive approximations of this character were standard practice in orbit determination for a great many years more than should have been the case! There were clumsy experiments with the "locus fictus" which is shown in Figure 2 as the intersection of the line of *T*☄ with the line *ES*. When Gibbs became interested in the orbit problem (1889),[2] largely in connection with his development of vector analysis, he was fortunately ignorant of astronomical practice. Consequently he decided very simply to correct the solar coordinates or the vector R from the center of the earth to the observer, at the start of the problem, by subtracting the known vector r_T, thus replacing the triangle ES☄ by the triangle TS☄. We find in the literature that this thought had occurred previously to Challis (1848),[3] and possibly to Leverrier (1855),[4] but had not taken hold. In fact astronomers were slow to adopt Gibbs' simple solution to the parallax problem until the much more recent contributions of Bower (1922, 1932),[5] Merton (1925),[6] Rasmusen (1951),[7] and others.

My own contribution to revised thinking in this area is associated with my work on my thesis[8] in 1935 and 1936 and with a mathematically oriented contribution of Poincaré (1906).[9]

Poincaré had suggested a "second approximation" for the Laplacian method of determining orbits. In the Laplacian method three observations of α and δ are numerically differentiated in order to produce velocities and accelerations in these angular coordinates (see Figure 3). The numerical differentiation ignores the higher derivatives in the first approximation, and it was these that Poincaré aimed to restore in his "second approximation." The Laplacian solution usually involves an assumption that the observer is travelling in a two-body orbit, and this assumption was uncritically accepted by Poincaré. But it is not the observer (T in Figure 3) who travels in a two-body orbit, nor is it even the center of the Earth (E in Figure 3) but (to a high degree of approximation) it is the barycenter of the Earth-Moon system (B in Figure 3). Williams (1934)[10] attempted to make the Poincaré method work by correcting for geocentric parallax, but found that barycentric parallax ultimately prevented the process from converging. He did not attempt to apply Leuschner's (1913)[11] technique for complete elimination of parallax. William's work came to my attention when I was writing my thesis.

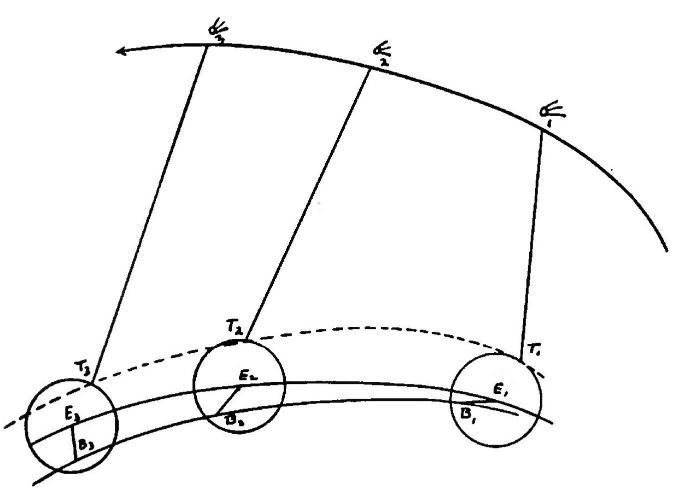

FIGURE 3.—Illustration of real problems of orbit determination.

In reviewing the matter I became aware that the "motion of the observer" has nothing whatsoever to do with the problem, but is only a mathematical fiction: the "observer" may actually be three different observers at three different observatories. Consequently I decided to assume that this fictitious motion is determined by the real motion of the object and by the further assumption that the higher derivatives of the observed angular coordinates were zero. These assumptions made it possible to carry the "second approximation of Poincaré" to a successful "real" conclusion.

These assumptions also made it possible to relate the basic first approximation of the Laplacian methods exactly to the first approximation in the methods of Gauss,[12] Lagrange,[13] and Gibbs,[14] a relationship that is necessary to the development of criteria for the selection of method in "real" problems of orbit determination.

Linearization in Astrodynamics

One of the issues in astrodynamics that is still unresolved nearly three decades after 1939 is the use of linear methods in astrodynamics. Many linear methods based upon the work of Poincaré have been brought back into celestial mechanics without realization on the part of Poincaré or his successors that non-linear solutions to the problems considered not only exist, but have been in constant use! Nevertheless some of the ideas have been provocative, and newer uses may be found for them.

It seems clear at present that linear methods may be used *after* a basic non-linear integration is complete, especially to obtain partial derivatives, but that their use in the basic integration is suspect, and may be either erroneous or unnecessary or both.

The basic geometrical equation used in the comparison of a theory with observations is certainly in a category for which linearization is allowable, and I find that Stumpff (1931)[15] and I (1940)[16] were experimenting in the use linear combinations of the residuals before 1940. The equation is

$$\rho L=\rho=r+R \qquad L=\begin{Bmatrix}\cos\delta\cos\alpha\\ \cos\delta\sin\alpha\\ \sin\delta\end{Bmatrix}$$

where r is SE in Figure 2 and R has been corrected from ES to TS as discussed above.

Stumpff had already proposed the use of residuals in two of the ratios between the three components of L, selected according to size, when I, having proposed residuals in the interdependent components themselves, realized the equivalence of the two proposals. Essentially their aim was the avoidance of successive trigonometric recalculations of α and δ in comparisons of successive theories with the observations.

The basic Stumpff concept, I found, could be extended to residuals in ρ or r or even to the "ratios of the triangles" used in preliminary orbit determinations by the methods of Lagrange, Gauss, and Gibbs.

Series Expansions

Preliminary orbit determination, perturbation theory, correction theory, all make effective use of series expansions of many kinds. The use of Fourier series (or epicycles) has been remarked upon in the foregoing. Power series now almost universally called the "f and g series" were developed by Lagrange (1783) [17] for the equations

$$r_j=f_jr_0+g_j\dot{r}_0$$

and from the series for $j=1$, 3 (with 0 replaced by 2) were developed the series for the "ratios of the triangles" referred to above. Gibbs (1889) [18] reexamined these expansions with his usual clear-sightedness and contributed new expressions for the "ratios" that have been the most generally recognized of his contributions to orbit theory. Happily, he left for me (1940) the extension of his developments to companion expressions, even simpler, for the determination of velocity components from three sets of position components.[19] These expressions have made the Lagrangian method for determining a preliminary orbit as effective as the Gaussian, but simpler. They enter also into orbit determinations that involve modern electronic observations of "range-rate."

In Conclusion

The foregoing remarks have been designed to give not a complete history of the pre-1940 foundations of astrodynamics, but rather samplings of these foundations that reveal the character of the subject, as it may be partially distinguished from the more purely mathematical developments of celestial mechanics. These samples nevertheless demonstrate again that universal principles and ideas tend to crop up independently in more than one time or place, that their excellence depends upon provability, and that they will be used when the time is ripe if they are continuous from sound antecedents.

Subsequent decades were to build enormously on the pre-1940 foundations, and to expand them, in conjunction with new instrumentation, with new vehicles, and with searches for previously inaccessible physical constants or for greater accuracy in relativity constants, the solar parallax, and other basic data of value both to physics and to precision space navigation.

NOTES

On 21 March 1974 Dr. Samuel Herrick Jr. died. His obituary was carried in *The Washington Post* of 25 March 1974. —Ed.

1. Samuel Herrick, *Astrodynamics* (London, New York: Van Nostrand Reinhold, 1971), vol. 1.
2. Josiah Willard Gibbs, "On the Determination of Elliptic Orbits from Three Complete Observations," *Memoirs of the National Academy of Science*, vol. 4, 1889, pp. 81–104; Ernst Friedrich Wilhelm Klinkerfues, *Theoretische Astronomie*, ed., edited by Hugo Buchholz (Braunschweig: Vieweg, 1912), pp. 413–18; and *The Collected Works of J. Willard Gibbs* (New York: Longmans Green, 1928), vol. 2, pt. 2, pp. 118–48.
3. James C. Challis, "A Method of Calculating the Orbit of a Planet or Comet from Three Observed Places," *Memoirs of the Royal Astronomical Society*, vol. 14, 1848, pp. 59–77.
4. Urbain Jean Joseph Leverrier's anticipation of the correlation of solar coordinates to the observer was once shown to the author by Ernest C. Bower, but he has not been able to find it for reference in this paper.
5. Ernest C. Bower, "On Aberration and Parallax in Orbit Computation," *Astronomical Journal*, vol. 34, 1922, pp. 20–30; and "Some Formulas and Tables Relating to Orbit Computation and Numeric Integration," *Lick Observatory Bulletin*, no. 445, vol. 16, 1932, pp. 34–45.
6. Gerald Merton, "A Modification of Gauss's Method for the Determination of Orbits," *Monthly Notes* (of the Royal Astronomical Society), vol. 85, 1925, pp. 693–731; ibid., vol. 86, 1926, pp. 150–51; ibid., vol. 89, 1929, pp. 451–53. Also

see Russell Tracy Crawford, *Determination of Orbits of Comets and Asteroids* (New York: McGraw-Hill, 1930), pp. 103–35.

7. Hans Qvade Rasmusen, "Tables for the Computation of Parallax Corrections for Comets and Planets," *Publikationer og mindre Meddelelser fra Københavns Observatorium,* no. 155, 1951, pp. 3–7.

8. Herrick, "The Laplacian and Gaussian Orbit Methods," *University of California Publication, Contributions of Los Angeles Astronomical Department,* vol. 1, 1940, pp. 1–56.

9. Jules Henri Poincaré, "Sur la détermination des orbites par la méthode de Laplace" [On the Determination of Orbits by the Method of Laplace], *Bulletin Astronomique,* vol. 23, 1906, pp. 161–87.

10. Kenneth P. Williams, *The Calculation of the Orbits of Asteroids and Comets* (Bloomington: Principia, 1934).

11. Armin Otto Leuschner, "Short Methods of Determining Orbits" (Second and third papers), *Publication of the Lick Observatory, University of California,* vol. 7, 1913, pp. 217–376 and 455–83.

12. Carl Friedrich Gauss, *Theoria motus corporum coelestium in sectionibus conicis solem ambientium* [Theory of the Motion of the Heavenly Bodies Moving about the Sun in Conic Sections] (Hamburg, 1809); translated by Charles Henry Davis (Boston: Little, Brown, 1857).

13. Joseph Louis Lagrange (1736–1813), "Sur le problème de la détermination des orbites des comètes, d'après trois observations" [On the Problem of the Determination of the Orbits of Comets from Three Observations], *Nouvelle Mémoire de l'Academy Royale des Sciences et Belles-Lettres de Berlin; Oeuvres* (Paris: Gauthier-Villars, 1869), vol. 4, pp. 439–532.

14. See note 2.

15. Karl Stumpff, "Uber eine kurze Methode der Bahnbestimmung aus drie oder mehr Beobachtungen" [On a Short Method of Orbit Determination from Three or More Observations), *Astronomischer Nachrichten,* vol. 243, 1931, pp. 317–36, and vol. 244, 1932, pp. 433–64.

16. See note 8.

17. See note 13.

18. See note 2.

19. See note 8.